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Series parallel transformation of impedances

Any series connection of an inductor or capacitance with an ohmic resistor can be replaced by an equivalent parallel circuit (and vice versa) if only a fixed frequency is used.
Both circuits show the same terminal behavior (but only at this frequency!).

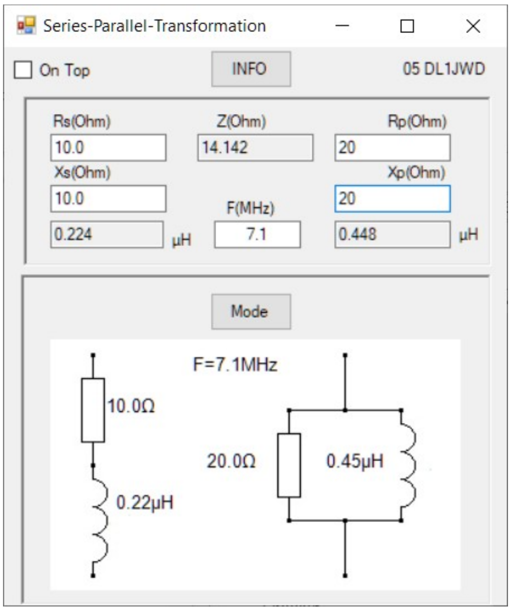
In many cases, meaningful series parallel transformations can significantly simplify the analysis of AC circuits (e.g. passive filters).

The tool uses the units of measurement MHz, µH and pF commonly used in amateur radio.

Example 1:

At the input of the antenna feed line I measure the impedance (e.g. with a VNA)
Z(ohm) = 10 + j10.
How large are the real and reactive parts of the equivalent parallel connection?
How big are the inductors at a frequency of 7.1MHz?

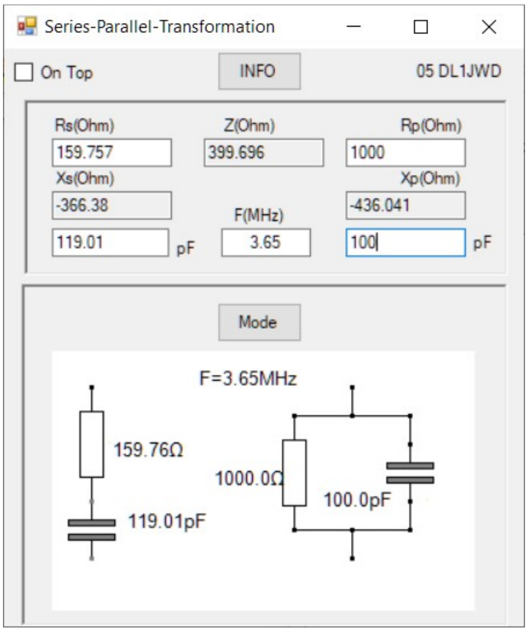
After entering Rs and Xs, you can immediately calculate the magnitude of Z (14.142Ohm) and the values of the equivalent equivalent circuit diagram Z(Ohm) = 20 || j20.
If you enter the frequency, the corresponding inductance values (0.22µH or 0.45µH) appear in the grey fields below.



Example 2:

A capacitance of 100pF has a resistor of 1kOhm connected in parallel.
What does the equivalent series connection look like at a frequency of 3.65MHz?

Click several times on the "Mode" button until the edit fields for capacities are released at the top. First enter the frequency and then the values for parallel resistance and parallel capacitance on the right.
The data of the equivalent series connection (160 ohms and 120pF) are immediately readable.



Theory

Prerequisite for understanding the transformation series connection = > parallel connection is some basic knowledge of calculating with complex numbers (also called phasors in electrical engineering).

Between a complex resistor (impedance)

$$\boldsymbol{Z}=R+jX$$

and a complex conductance (admittance)

$$\boldsymbol{Y}=G+jB$$

is the relationship:

$$\boldsymbol{Y}=\frac{1}{\boldsymbol{Z}} \quad \text{bzw.} \quad G+jB=\frac{1}{R+jX}$$

To get the j-operator out of the denominator, we extend the numerator and denominator of the right side of the equation with the factor R-jX:

$$G+jB=\frac{R-jX}{(R+jX)(R-jX)}$$

After multiplying and splitting into real and imaginary parts, the result is:

$$G+jB=\frac{R}{R^2+X^2}-\frac{jX}{R^2+X^2}$$

Since real and imaginary parts must match on both sides of the equation, we obtain the following relationships for the conversion of an impedance into the equivalent admittance:

$$G=\frac{R}{R^2+X^2}$$

$$B=\frac{-X}{R^2+X^2}$$

For the determination of the parallel connection Rp || jXp at a given series connection Rs+jXs we set Rp = 1 / G and Xp = -1 / B and finally get:

$$R_p=\frac{R_s^2+X_s^2}{R_s}$$

$$X_p=\frac{R_s^2+X_s^2}{X_s}$$

For the reverse transformation (parallel connection = > series connection) we take the opposite approach

$$R+jX=\frac{1}{G+jB}$$

and arrive at the relationships via a similar calculation path:

$$R_s=\frac{R_p}{1+\frac{R_p^2}{X_p^2}}$$

$$X_s=\frac{X_p}{1+\frac{X_p^2}{R_p^2}}$$

For the calculation of capacitive and inductive reactances, the following tailored size equations are suitable:

$$X_L[k\Omega]=\omega[GHz]L[\mu H]$$

$$X_C[k\Omega]=\frac{-1}{\omega[GHz]C[pF]}$$

$$\text{with}\quad \omega=2\pi f$$