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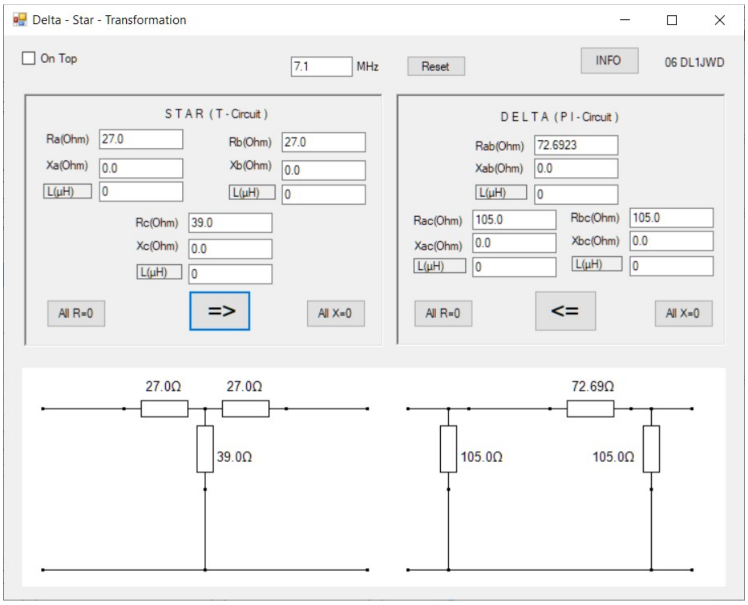
Star-delta transformation of impedances

A T-circuit (star) can be replaced by a Pi circuit (delta) with the same terminal behavior and vice versa.
The calculation becomes much more complicated when it is not ohmic, but complex resistances (impedances). These can also have negative real parts after the transformation, they are then technically not feasible.

Example 1:

An attenuator (-9.72dB, 50Ohm system) in T-circuit consists of the resistors Ra = 27Ohm, Rb = 27Ohm and Rc = 39Ohm. What does the equivalent Pi circuit look like?

Since all resistors are frequency-independent, the entered value for the frequency does not matter.
Click the button "All X=0" on the left side, as this is a pure resistance network.
Fill in the input fields for Ra, Rb and Rc and click the button "=>" to execute the transformation.



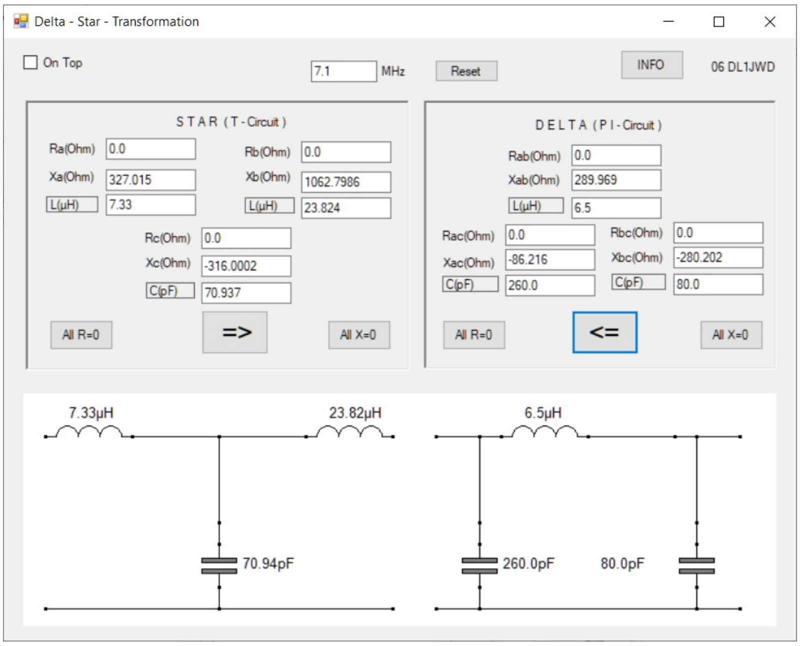
Example 2:

A Collins filter with C1 = 260pF, C2 = 80pF and L = 6.5µH used for antenna matching at 7.1MHz is to be transformed into the equivalent T-circuit.

Click the button on the right "All R=0".
After entering the frequency, enter the values for L and C of the delta connection on the right half.

To switch between inductance and capacitance, you must first click on the small framed fields "L(µH)" or "C(pF)"!

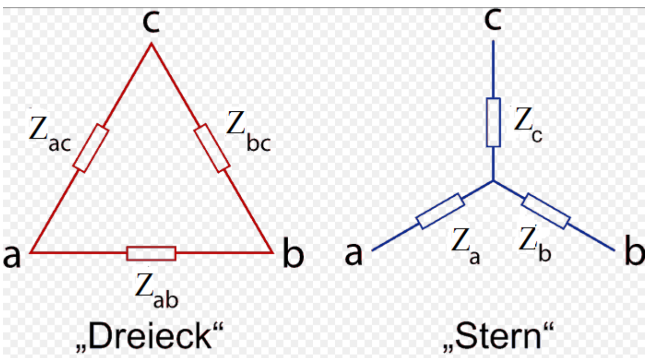
Finally, click the button "<=" to execute the transformation to the T-circuit.



So far, we have neglected the losses in components L and C.
We now want to change this by adding a series resistor to the coil of the Collins filter.
Rab = 3Ohm, which corresponds to a coil quality of about 100.
Unfortunately, you will find that this time there is no exactly equivalent T-circuit, since its resistance Rc should get a negative value.

Theory

The star-delta or triangle-star transformation is a delta-star transformation or Kennelly theorem named after Arthur Edwin Kennelly.
By applying both transformations and the rules for parallel and series connection of impedances, the analysis of AC circuits (this includes the special case of pure resistance networks, of course) can be simplified.
For example, a star-delta transformation is synonymous with eliminating the inner node of the circuit.
The star-delta transformation is identical to the Pi-T transformation between the π circuit and the T circuit.



In both circuits, the impedances measured between terminals a, b and c must be exactly the same.
This condition results in the following conversion formulas, the derivation of which, however, is only recommended to those who are reasonably well versed in complex arithmetic operations (phasors):

Star => Delta:

$$Z_{ab} = \frac{Z_a Z_b + Z_a Z_c + Z_b Z_c}{Z_c}$$

$$Z_{ac} = \frac{Z_a Z_b + Z_a Z_c + Z_b Z_c}{Z_b}$$

$$Z_{bc} = \frac{Z_a Z_b + Z_a Z_c + Z_b Z_c}{Z_a}$$

Delta => Star:

$$Z_a = \frac{Z_{ac} Z_{ab}}{Z_{ac} + Z_{ab} + Z_{bc}}$$

$$Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ac} + Z_{ab} + Z_{bc}}$$

$$Z_c = \frac{Z_{ac} Z_{bc}}{Z_{ac} + Z_{ab} + Z_{bc}}$$

It applies to the impedances:
Za = Ra + jXa; **Zb** = Rb + jXb; **Zc** = Rc + jXc
Zab = Rab + jXab; **Zac** = Rac + jXac; **Zbc** = Rbc + jXbc

If it is pure resistance networks, then all blank components (all X) are zero, which of course leads to a considerable simplification of the conversion.