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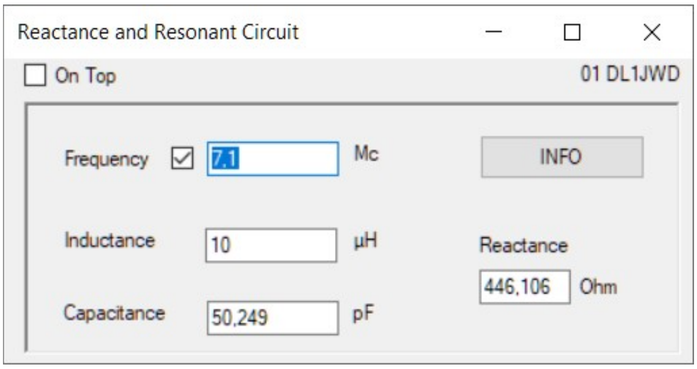
Reactance and Resonant Circuit

The determination of capacitive or inductive reactances as well as the resonant frequency of resonant circuits are part of the basics of the experimental radio amateur.
The tool uses the usual shortwave units MHz, µH and pF.

After zooming in on the window, after entering the quality factors of the coil and capacitor, resonance resistance, operating quality, 3dB bandwidth and volume attenuation can also be analyzed for different configurations of series and parallel resonant circuits.

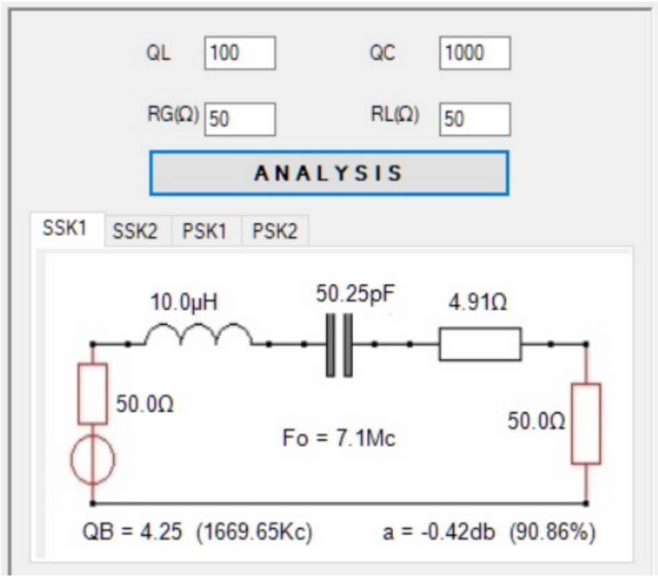
Example 1:
What is the reactance of an inductor of 10µH at 7.1MHz?
How large does the capacitance have to be to form a resonant circuit?



With the check mark in front of the input field for the frequency it is achieved that not the frequency, but the capacitance is recalculated.
The reactance appears synchronously with the change of L or C, without you having to click a result button.

However, without changing the frequency you can also enter a different reactance to make the resonant circuit impedance higher or lower and calculate the required LC pair.

Example 2:
The resonant circuit is to be operated as a suction circuit between a voltage source with 50Ω internal resistance and an equal load resistor.
What is the resonant resistance, bandwidth and transmission attenuation if the coil quality is 100 and the quality of the capacitance is 1000?

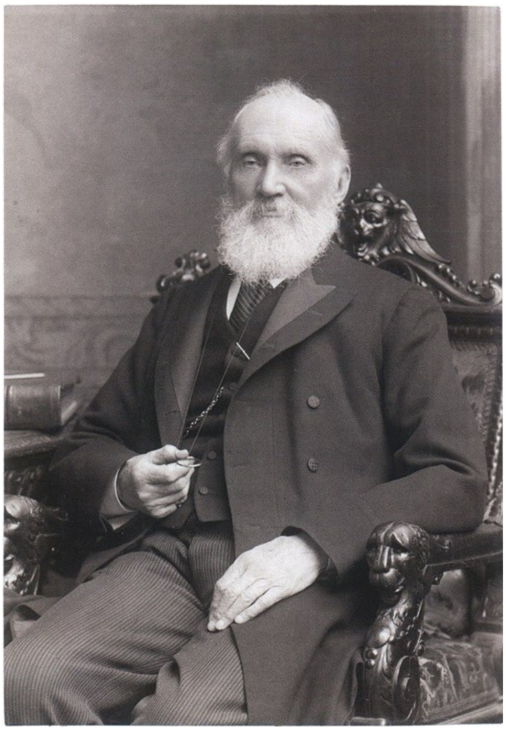


Enlarge the window by touching it with the mouse at the bottom of the window and zooming in. Enter the values for the quality-factors and for the terminating resistors.

Click on "ANALYSIS" and select the configuration "SSK1" (series resonant circuit) to read the results in the schematic.
In brackets, the 3dB bandwidth (kHz) is printed behind the operating mode QB and the transmission (%) behind the transmission loss a (dB).
Change the reactance (or L/C ratio) and observe the effects on 3dB bandwidth and transmission loss!

Theory

The basis of this fundamental tool is the well-known "Thomson's vibrational equation", as it was first formulated in 1853 by the brilliant British physicist William Thomson.
Also the unit of measurement of absolute temperature (degrees Kelvin) also goes back to Thomson, because as a professor of theoretical physics in Glasgow he was knighted for his outstanding services in the fields of electricity theory and thermodynamics and raised to the hereditary peerage as Baron Kelvin.



The starting point of Thomson's discovery was the realization that inductors and capacitances also oppose the electric current, which, however, – in contrast to ohmic resistance – is frequency-dependent and does not convert any electrical energy into heat (hence reactance).

The **reactance of an inductor L** at frequency f is calculated as:

$$X_L = \omega_0 L \quad \text{mit} \quad \omega_0 = 2\pi f_0$$

or in our usual units of measurement:

$$X_L[\Omega] = 6{,}28 f_0[MHz] L[\mu H]$$

For the **reactance of a capacitance C**:

$$X_C = \frac{-1}{\omega_0 C}$$

$$\text{or} \quad X_C[\Omega] = \frac{-159236}{f_0[MHz] C[pF]}$$

In the **case of resonance**, both reactances compensate each other:

$$X_L + X_C = 0 \quad \text{bzw.} \quad \omega_0 L - \frac{1}{\omega_0 C} = 0$$

The above equation broken down by ω_o gives $\omega_0 = \frac{1}{\sqrt{LC}}$

or with $\omega_0 = 2\pi f_0$ the best-known form of **Thomson's resonant equation**:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or

$$f_0[MHz] = \frac{10^3}{2\pi\sqrt{L[\mu H] C[pF]}} = \frac{159{,}236}{\sqrt{L[\mu H] C[pF]}}$$

In practice, inductors and capacitances also have ohmic loss resistances r_L or r_C, whose ratio to the reactant resistors X_L or X_C is expressed by the quality factors Q_L and Q_C.

For an inductor with the series loss resistance r_L, the **coil quality** is:

$$Q_L = \frac{\omega_0 L}{r_L}$$

The same applies to the **quality of a capacitance** with the series loss resistance r_C:

$$Q_C = \frac{1}{\omega_0 C r_C}$$

The **operating mode quality-factor QB** of an resonant circuit depends on the reactance (L/C ratio) and on r_L and r_C, but also on the damping influence of the generator resistance R_G and the load resistance R_L.

With $$X_L = X_C = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}}$$

applies to the series resonant circuit:

$$Q_B = \frac{\sqrt{\frac{L}{C}}}{r_L + r_C + R_G + R_L}$$

A similar formula could be given for the parallel resonant circuit, but the serial loss resistances must first be transformed into their parallel equivalents.

The operating mode quality-factor Q_B allows direct conclusions to be drawn about the selection properties of an resonant circuit, so the following applies to its 3dB bandwidth:

$$B_{3dB} = \frac{f_0}{Q_B}$$

With increasing operating quality, the bandwidth decreases, i.e. the resonant circuit becomes narrower.
At the same time, however, the losses increase, as the transmission (power transmission) is also reduced.
The transmission of a two-pole (e.g. resonant circuit) connected between generator resistance R_G and load resistor R_L is calculated from the ratio of the power converted at R_L to the maximum available generator power:

$$vp = 4 |v_U|^2 \frac{R_G}{R_L}$$

where $|v_U|$ the voltage gain, i.e. the amount of the ratio between the voltage at the load resistor R_L and the generator voltage U_0:

$$v_U = \frac{U_{R_L}}{U_0}$$

The **transmission attenuation** results from the decadal logarithm of the transmission:

$$a[dB] = 10 \log(v_p)$$